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Degree-III(H) Paper-V, Group-C, 19/02/2024.

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Topic:- Uncertainty Principle

Derivation:- Consider a system in a quantum mechanical state $|\psi\rangle$ and imagine measuring two observables represented by two Hermitian operators A and B respectively. For simplicity, we will assume that the spectra of both operators are discrete with.

$$A|a_i\rangle = a_i|a_i\rangle \text{ and } B|b_j\rangle = b_j|b_j\rangle \quad \text{— (1)}$$

Clearly, the measurement of A yields as the result one of the eigenvalue, a_i and if we expand the state of the system in terms of the eigenkets of A. We have,

$$|\psi\rangle = \sum a_i|a_i\rangle \text{ where } a_i = \langle\psi|a_i\rangle \quad \text{— (2)}$$

The results of the measurement of A yield a distribution of values

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with the mean given by.

$$\langle A \rangle \equiv \langle \psi | A | \psi \rangle = \sum_i |a_i|^2 a_i \quad \text{--- (3)}$$

Similarly, one can derive $\langle A^2 \rangle$ and this enables one to define σ^2_A for this state for the observable A as usual by

$$\sigma^2_A = \langle \psi | A^2 | \psi \rangle - \langle \psi | A | \psi \rangle^2.$$

This is defined to be the uncertainty in the measurement of A. We will derive a constraint on the simulta-neous measurement of two observables by computing the uncertainties in measuring A and B on the same state $|\psi\rangle$.

$$\because \quad \Delta A = A - \langle A \rangle$$

And $\quad \Delta B = B - \langle B \rangle$

where $\langle A \rangle = \langle \psi | A | \psi \rangle$ as usual.

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Consider.

$$|\phi\rangle = (\Delta A - i\lambda \Delta B)|\psi\rangle$$

$$\langle\phi|\phi\rangle \geq 0$$

Now $\langle\psi|(\Delta A + i\lambda\Delta B)(\Delta A - i\lambda\Delta B)|\psi\rangle \geq 0$

$\because$ We have,

$$\sigma^2 A = \langle\psi|(\Delta A)^2|\psi\rangle$$

and $\sigma^2 B = \langle\psi|(\Delta B)^2|\psi\rangle$

We obtain the following upon substitution

$$\sigma^2 A + \lambda^2\sigma^2 B - i\lambda\langle\psi|[\Delta A, \Delta B]|\psi\rangle \geq 0$$

$$\because [\Delta A, \Delta B] = [A, B]$$

Again $\lambda^2\sigma^2 B + \lambda\langle C\rangle + \sigma^2 A \geq 0$

$$\therefore \langle C\rangle^2 \leq 4\sigma^2 A\,\sigma^2 B$$

Substituting the value of C we get

$$\sigma^2 A\,\sigma^2 B \geq \left\langle \frac{1}{2i}[A, B]\right\rangle^2$$

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For $A = x$ and $B = p_x$ we obtain,

$$\boxed{\sigma_x \sigma_p \geq \frac{\hbar}{2}}$$

Above Equ is the standard form of the Heisenberg uncertainty relation.

Note :- If two operators commute, for example, x and p_y, the x-coordinate and the y-component of the momentum there are no restrictions on the arbitrarily accurate, simultaneous measurement of the two observables, in principle.